

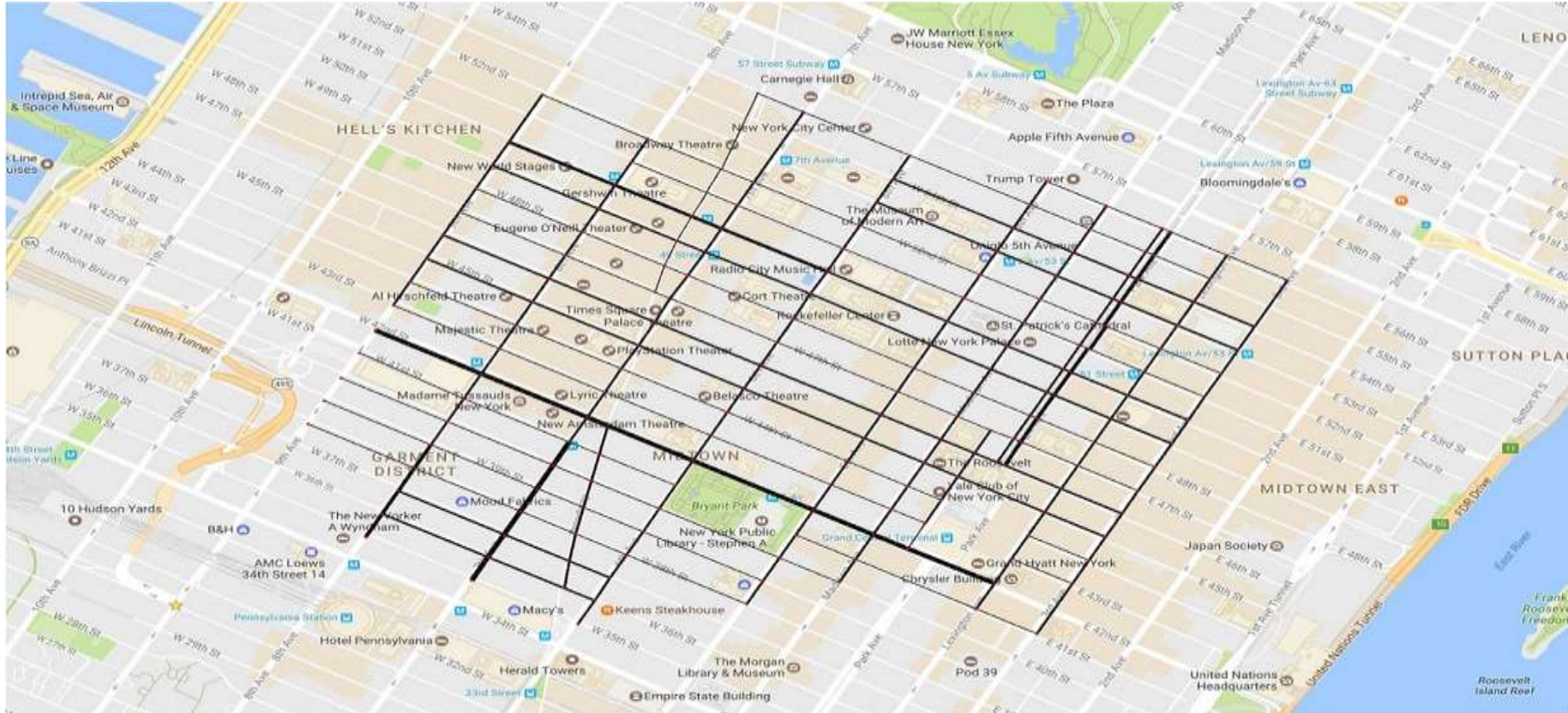
# Compositional and Contract-based Verification for Autonomous Driving on Road Networks

**Lucas Liebenwein**, Wilko Schwarting, Cristian-Ioan Vasile,  
Jonathan DeCastro, Javier Alonso-Mora, Sertac Karaman, Daniela Rus

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# How can we obtain safety guarantees?



# Problem Definition



# Problem Definition

Verification Components		
System	Model	Specification – Controller Contract $\mathcal{S}$
<ul style="list-style-type: none"><li>• <b>Controller:</b> <math display="block">\mathbf{u}_k = C(\mathbf{z}_k^{0:N}),</math>where <math>\mathbf{z}_k^i</math> is state of car <math>i</math> at time <math>k</math> with  <b>The controller is assumed to abide by the controller contract <math>\mathcal{S}</math>.</b></li></ul>		

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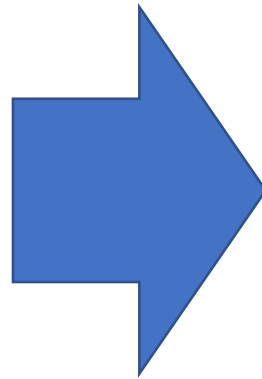
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# Technical Challenges

- Verification is HARD
- Computational tractability often not feasible
- Model complexity increases cost
- Realistic scenarios requires scalable models

# Technical Challenges

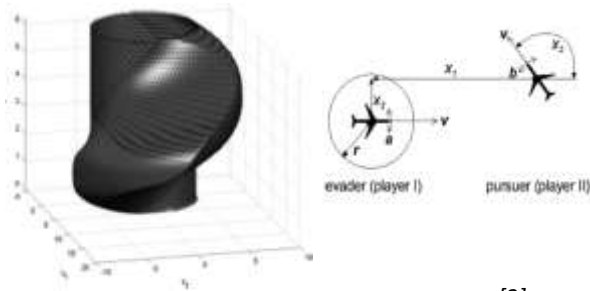
- Verification is HARD
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- Realistic scenarios requires scalable models



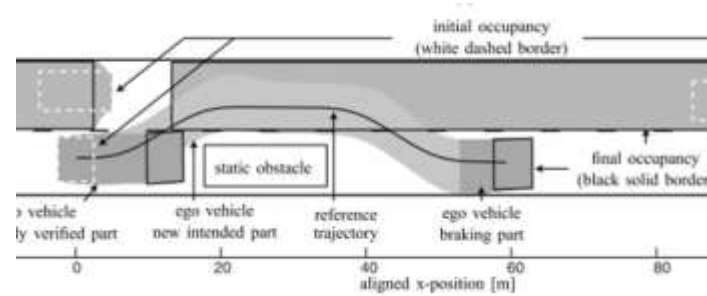
How do we maintain computational tractability while capturing realistic scenarios?



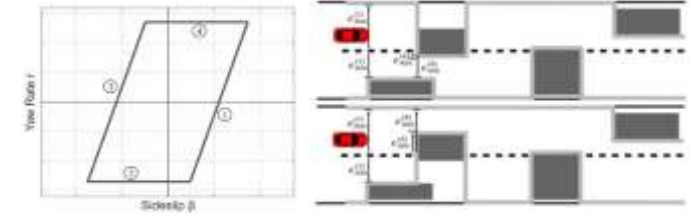
# Related Work



Mitchell, I.M., et.al.<sup>[2]</sup>



Althoff, M., et.al.<sup>[3]</sup>



Erlie, S.M., et.al.<sup>[4]</sup>

- Backwards Reachability from Goal
- Formulation as Hamilton-Jacobi PDE
- Solution over Discretized State Space

- Online Verification of Planned Maneuvers
- Rough Approximations for Reachable Sets
- Safe Backup Trajectory

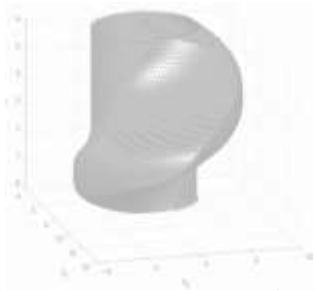
- Shared Steering Control
- Safety Guarantees through Dynamical and Road Constraints
- MPC Looks for Possible Trajectory to Ensure Safety

[2] Mitchell, I.M., et. al.: A time-dependent Hamilton-Jacobi formulation of reachable sets for continuous dynamic games. IEEE Transactions on Automatic Control 50(7), 947–957 (2005)

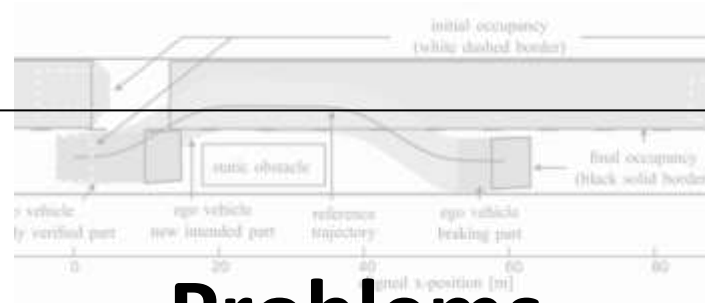
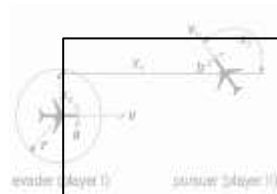
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## Problems

Althoff, M., et.al.<sup>[2]</sup>



Erlie, S.M., et.al.<sup>[3]</sup>

- Backwards Reachability from Goal
- Formulation as Hamilton-Jacobi PDE
- Solution over Discretized State Space

- **Accurate but very specific**
- **More general but low-fidelity**

- Online Verification of Planned Maneuvers
- Rough Approximations for Reachable Sets
- Safe Backup Trajectory

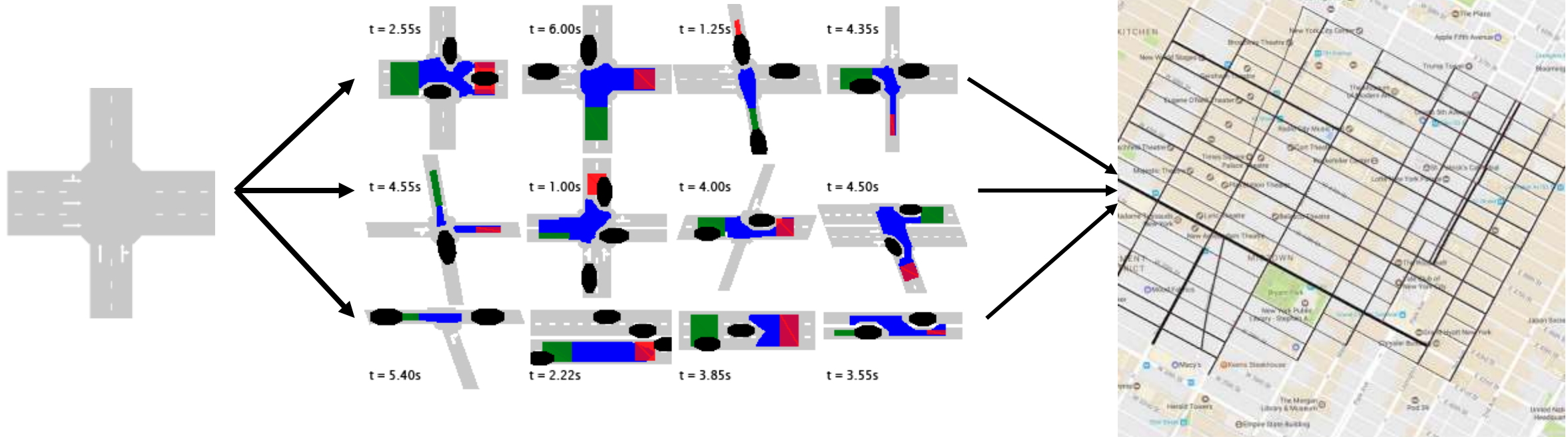
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# Two Step Approach

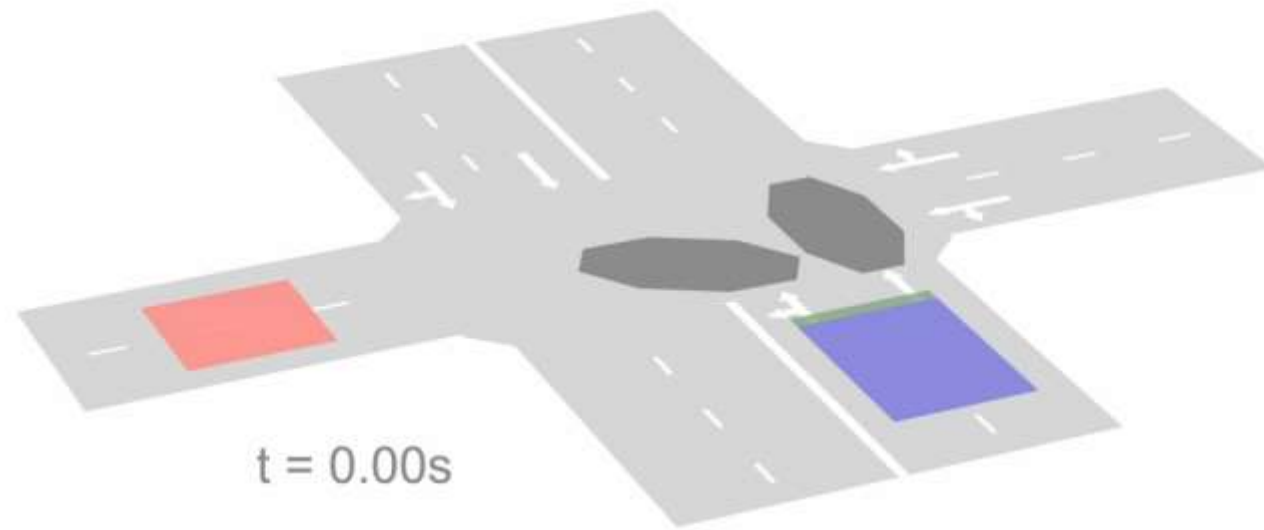





Library of Road Geometries

Parallel Local Verification

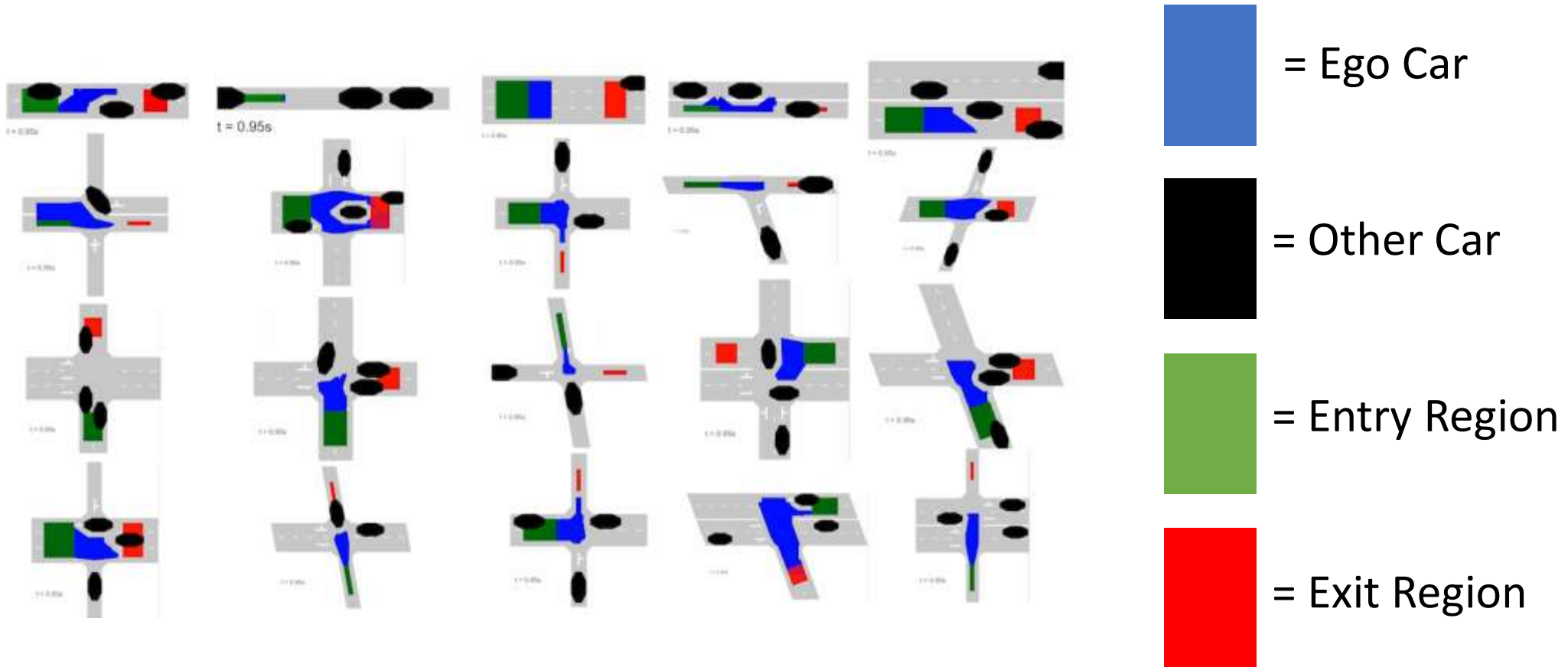
Global Guarantees

# Local Verification – Results



-  = Ego Car
-  = Other Car
-  = Entry Region
-  = Exit Region

# Local Verification – Results



# Global Guarantees – Results



= Ego Car



= Other Car



= Entry Region



= Exit Region

# Global Guarantees – Results



= Ego Car



= Other Car



= Entry Region



= Exit Region



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